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HEAT FLUX DETERMINATION USING SURFACE AND BACKFACE TEMPERATURE HISTORIES AND INVERSE METHODS

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Abstract

The objective of this study is to develop a small, robust, accurate and cost-effective gage that can be used to measure surface heat flux and that can be combined with a skin friction sensor. The method adopted is based on a fast-response, coaxial surface thermocouple with an additional thermocouple to measure the temperature on the back face of the test article. The surface heat flux is determined by an inverse method using these two temperature histories. This alleviates the often restrictive semi-infinite body assumption common in other methods. To prevent lateral conduction effects, the gage is manufactured from a material similar to the test article. Therefore, a one-dimensional approach was followed, but a study of two-dimensional effects was necessary to verify the validity of the one-dimensional assumption. The inverse approach is capable of predicting the heat flux through a surface from the measured surface temperature history. One important advantage of this method is the passivation of noise inherent in the system. As part of this study, the amplification of noise induced by the Cook-Felderman technique was investigated in detail. Heat flux estimates from the Cook-Felderman technique as well as the inverse technique compared well with measurements recorded with a commercially-available layered heat flux gage for a test case of a model turbine blade in a linear cascade. However, it was found that the inverse approach does not amplify measurement noise and provides better estimates. The results from the inverse method are somewhat noisier than those from the commercial heat flux gage, but the size of the current gage is much smaller when using a coaxial thermocouple to measure the surface temperature history. A coaxial thermocouple gage is also much more robust and cheaper and simpler to manufacture than the other types of heat flux sensors.

Nomenclature

q	heat flux (W/m^2)
k	thermal conductivity (W/m K)
t	time (sec)
α	thermal diffusivity (m^2/s)
β	regularization parameter
ρ	density (kg/m^2)
C_p	specific heat capacity (J/kg K)
T	temperature(K)

Introduction

The ultimate goal of this work is to install a heat flux measuring device into a skin friction gage of Virginia Tech design[1] for use in hot, high-speed flows of

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durations in seconds. In the past, this device measured only skin friction, and the addition of heat flux measuring capabilities will enhance the overall performance and capabilities of the instrument. Before determining which type of heat flux gage to employ, it was important to consider the current most popular and widely used gages.

The measurement of convective heat flux became very popular in the 1950's. Since then, numerous different heat flux gages have been developed, and literally hundreds of reviews have been written on the measurement of heat fluxes.[2,3] Convective heat flux measurements and gages can be classified into four broad categories as follows:

- A temperature difference can be measured over a spatial distance with a known thermal resistance.
- A direct measure of the energy input or output can be obtained.
- The flux can be obtained by measuring the temperature gradient in the fluid adjacent to the surface.
- A temperature (or temperature difference) can be measured as a function of time.

Layered gages are the simplest of the spatial temperature difference gages.[2] The temperature is measured on either side of a thermal resistance layer. The temperature difference is then proportional to the flux into or out of the surface. These gages vary in size, materials and in the way the temperatures are measured. The sensitivity of these gages is not only a function of the temperatures being measured, but also of the thermal conductivity of the thermal resistance layer and the thickness of this layer. A drawback of these gages is the fact that they can have relatively slow response times, especially if the resistance layer is thick. A *wire-wound gage* can be used to overcome the problem of time response. Wire wound gages consist of one of the thermocouple wires (say Constantan) wrapped around a thermal resistance layer. One half of the wire is then electroplated with the other thermocouple material, e.g. Copper. This creates thermocouple junctions on the top and at the bottom of the thermal resistance layer. The thermal resistance layer is usually made of a very high thermal conducting material. The sensitivity of these gages is good, but one of the drawbacks is the fact that one-dimensional heat transfer is not maintained. The *in-depth-temperature gages* are the third type in this category. The in-depth temperature gage measures the temperature in the wall itself, at two known positions, with a known distance between them.

The next category of heat flux gages is termed *active heating*. Active heating involves the measurement of the energy flow and then relating that to the flux into and out of the body. This energy is supplied electrically and can be controlled easily and accurately. The energy input to the system can then be related to the flux at the boundary, because the rest of the body is insulated. Heating takes place inside the body, and this causes the drawback that the flux is always out of the body. Two important issues are the effect of the thermal capacitance of the wall material, (if steady state is not achieved) and the heat transfer at the end of the measuring region. Very good insulation is important. The time constraints in this method make it unsuitable for use in high-heat-flux or high-temperature situations.[2]

Another category involves the *measuring of the temperature gradient in the fluid* next to the wall. This can be done in various ways. Thermocouples probing the fluid at known distances will measure the temperature as a function of time. The flux in the fluid adjacent to the wall can be considered to be the same as that in the wall. This method was not considered in detail here, because it is not used very much in practice, and the applications are limited. It is clearly inappropriate in very hot flows. Probing the fluid also introduces an obstruction in to the flow, which might alter the flux.

The fourth and last category in the measurement of heat flux is the *temperature-change-with-time* category. In this approach, the aim is to measure the surface temperature and by means of a mathematical scheme deduce the heat transfer at the surface. The first gage to be discussed in this category is the *slug calorimeter.*[2] This is a device that measures the amount of energy absorbed as a function of time. One temperature measurement at the back represents the entire gage. This assumption can be made only if the thermal resistance of the gage is very low. If we assume that the gage is insulated, then the losses can be neglected. The time response of these gages is, in general, not good. The *null-point calorimeter* offers an improvement on this drawback.[3] It works basically in the same way as the slug calorimeter. The null-point calorimeter only makes use of a hole drilled in the back of the wall to install the thermocouple closer to the surface. By doing this, the transient delay is much shorter, and the response is much faster.

The *thin-skin method* is a method like the slug calorimeter that covers the entire body of the model. Thermocouples are attached to the backside of the skin to measure the temperature. The heat flux can be calculated at each location using the same technique as that used by the slug calorimeter. The main errors occur as a result of conduction down the thermocouple wires, heat lost through the back surface of the skin and transverse conduction along the skin. Advances in the fabrication of *thin-film gages* made them largely replace the thin-skin gages.[4,5] These gages can be made very thin (~ 1 μm), and this improves the response times of these gages greatly. One of the biggest drawbacks when using these gages for high-temperature flows is that the gages are exposed to the hot, high-speed flows, and they may encounter damage during the test runs. These gages measure the surface temperature in response to the surface heat flux. The data analysis is the same as that used for null-point calorimeters. The largest difference between the *thin-film* gages and the coaxial thermocouple and the calorimeter is that the *thin-film* gage is mounted on the surface of a homogeneous solid.[2] The other two gages measure the temperature on a non-homogeneous plug inserted into the solid.

The last gage in this category is the *coaxial thermocouple* type gage to measure the surface temperature as a function of time. See F*igure 1.* Coaxial thermocouples are much easier to fabricate than their calorimeter counterparts. These thermocouples can be made very small, which increases their sensitivity as well as their response time. The sensitivity and response time are increased due to the smaller thermal mass that needs to be heated to a specific temperature. A very important issue when using these types of thermocouples to measure heat transfer is to match the physical properties of the surrounding wall very closely. The product, $k\rho C_p$, of the thermocouple and that of the wall, should be similar.[2] This is possible when one considers the fact that thermocouples can be made of virtually any two metals. By using one metal of the same thermophysical properties as that of the wall as the tube and the other a very thin wire, the thermocouple as a system can be made to have physical properties very close to those of the wall.

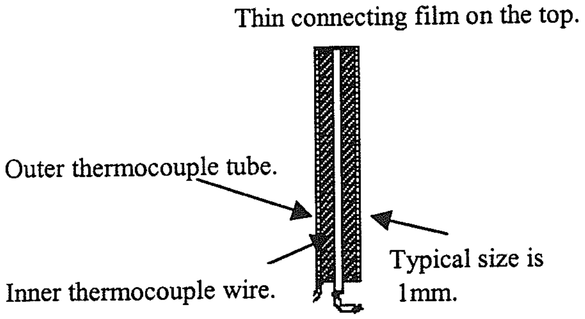

Figure 1. Coaxial Thermocouple for Surface Temperature Measurement.

In the present work, the surface temperature change with time type of method using a coaxial thermocouple was chosen after considering all the matters described briefly above. A data reduction scheme is then used to convert the measured temperatures to corresponding heat fluxes. This approach is often called the inverse heat conduction problem (IHCP). The inverse heat conduction problem is ill-posed, but has been well characterized.[6] Without the introduction of appropriate bias into the solution, noise in the measurements becomes amplified. When the measurements occur on the surface, the problem is not considered a true inverse problem, but it can be treated as one to alleviate the amplification of noise during the data reduction.

There are basically three different classes of methods to do the reduction of the data. The first class uses an analytical approach to the problem and is called the *Forward Analytical Method.* The second class approaches the problem in a numerical way, and is named the *Forward Numerical Method.* The third class makes use of an *inverse* technique. The inverse technique estimates a heat flux by minimizing the difference between calculated and measured temperatures. Using this inverse approach reduces some of the unwanted instabilities in the data reduction through the introduction of bias.[7]

Forward Analytical Methods: Class 1

The well-known Cook-Felderman technique[4, 8, 9] is one of the Class 1 techniques. The approach inverts the forward solution of the conduction equation assuming a semi-infinite body using the measured temperatures. The integration is performed assuming a piecewise linear temperature distribution between discrete measurements to yield:

$$q(t_n) = \frac{2\sqrt{k\rho C_p}}{\sqrt{\pi}} \sum_{j=1}^{n} \frac{T_j - T_{j-1}}{\sqrt{t_n - t_j} + \sqrt{t_n - t_{j-1}}} \qquad (1)$$

Diller and Kidd[3] undertook a modification to the Cook-Felderman technique to include future time steps which are used to prevent exaggeration of current estimates and stabilize the solution.

$$q(t_n) = \frac{\sqrt{k\rho C_p}}{\sqrt{\pi\Delta t}} \sum_{j=1}^{n} \left[\frac{T_j - T_{j-1}}{\sqrt{n+1-j}} + (T_{n+1} - T_n) + \frac{1}{3}(T_n - T_{n-1}) \right] \qquad (2)$$

For a detailed evaluation of these methods and their characteristics, refer to Ref. [10]. The semi-infinite body assumption is often a severe limitation in testing. We chose the Cook-Felderman method as one of the data reduction procedures considered here.

Forward Numerical Methods: Class 2

Class 2 techniques include fully numerical approaches developed to eliminate the primary drawbacks of the Class 1 problems, which include the restriction to constant properties. In Class 2 methods, the heat flux is obtained by using the derivative of the temperature profile at the surface. Differentiation of discrete data is inherently an unstable process,[11] and these methods suffer from amplification of measurement noise. As a result, these methods were not considered in the present work.

Inverse Methods: Class 3

The inverse heat conduction problems are Class 3 methods. In these problems, one is looking for the flux at the boundary, for example. A flux is guessed for each point in time, and this guessed flux is then used to calculate the surface temperature(s). This calculated temperature is compared to the actual measured temperature(s) on the surface(s), and the flux is then corrected until the measured and the calculated temperature(s) match to some predetermined error margin. The problem considered is not a true inverse problem, because the temperature in this case is measured on the surface. In a true inverse problem, the temperature is measured some distance below the surface and this temperature cannot be used as the boundary condition.[12]

For the present effort, we adopted the implementation of the inverse method of Ref. [13]. The objective function (equation 3) is a sum of squares of the residuals with added bias.

$$S = [Y - T(q)]^T \psi^{-1} [Y - T(q)] + \beta (Hq)^T (Hq) \qquad (3)$$

When the objective is minimized, the added bias has the effect of limiting the change in heat flux estimate between time steps. Therefore, the H matrix represents the first-order, finite-difference formulation coefficients.[14] T(q)'s are the calculated temperatures and Y's are the measured temperatures. The matrix of measurement variances (ψ^{-1}) is included as described by Beck.[6] β is the regularization parameter and is usually determined through trial-and-error to include enough bias to damp the noise, but not so much as to influence the solution. Determination of this prefactor will be discussed in a subsequent section. To solve for the heat flux, the temperature vector T(q) can be obtained with any appropriate forward conduction solution. If it is required to include temperature dependent properties, the solution will be non-linear. The solution will be formulated in terms of the flux correction Δq. This means that $q = q_o + \Delta q$, where q_o is the flux at the previous iteration and q is the new guess. Using a Taylor's series expansion of temperature and then equating to zero the differentiated objective function, a linear set of equations can be found. Δq can then be found from

$$\Delta q = [\mathbf{X}^T \Psi^{-1} \mathbf{X} + \beta \mathbf{H}^T \mathbf{H}]^{-1} [\mathbf{X}^T \Psi^{-1} (\bar{Y} - \bar{T}_0) - \beta \mathbf{H}^T \mathbf{H} q] \qquad (4)$$

Here, X is the sensitivity matrix that is defined as the derivative of the temperature with respect to the boundary flux. An iterative approach is used to calculate the flux correction (equation 4) based on an initial guess (this initial guess can be zero). The estimate is updated until the correction is arbitrarily small. To achieve biasing, one must consider future time steps. In this case, a first-order regularization method is used, and the minimum number of future times that can be used is two. More than two future time step evaluations will add too much bias.[15] The regularization determines how much bias is added, and an optimal value is determined by locating the point where the value of the objective is on the order of the measurement noise. A value of $\beta = 0.00001\ K^2m^4/W^2$ was found to work best.

Experimental Studies

Heat transfer measurement experiments were conducted in the Virginia Tech Transonic Cascade Wind Tunnel. The data collected were then reduced, and the corresponding heat fluxes were compared.

Test Facility and Model

Tests were performed in the Virginia Tech Transonic Cascade Tunnel. Details about the facility and the experiments to be discussed here can be found in Popp et. al.[16]

A 3-D view of one of the blades is shown in Figure 2. In Figure 3, the schematic layout of the two thermocouples installed in the blade is shown. The distance between the two thermocouples is 25mm.

Instrumentation

There were two Aluminum blades used in these experiments. The first blade was fully instrumented with six thermocouples, six Kulite pressure sensors, and six Vatell HFS heat flux gages.[10] The HFS gage consists of a number of thermocouples connected in series across a thin thermal resistance layer. The thermocouples in this blade were Type K. Another similar blade was used but with only one of each of the gages installed at the surface. In this blade, a coaxial thermocouple was installed in addition to the Type K thermocouples. This thermocouple was custom made by the Medtherm company. This thermocouple is not a conventional one, because it is made from Aluminum (Al) and Constantan. The reason Aluminum was used was to match the thermophysical properties of the Aluminum blade as closely as possible. This will ensure that the heat flux through the thermocouple itself will be as close as practically achievable to the actual flux through the body. Further, since the outer tube of the thermocouple, which is in contact with the Aluminum model, is also Aluminum, the system should be free of the extraneous EMF problems discussed in Ref. [17]. The blade is made of 6061 Aluminum alloy, and the thermocouple tube is made of 3003 Aluminum. The thermophysical properties of these alloys are given in Table 1.

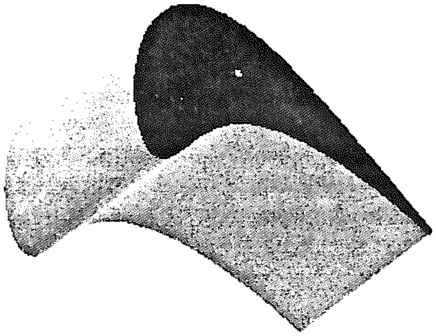

Figure 2. 3-D View of a Test Blade.

The basic concept of the thermocouple is illustrated in Figure 4. The thermocouple is made from a small Aluminum tube on the outside and a very thin Constantan wire on the inside. A thin ceramic insulator tube (Alumina) then separates the two conductors. The electrical connection on top of the thermocouple is made by depositing a thin film (≈1 μm) of Aluminum on the top. The thinner the connecting layer, the faster the response time of the thermocouple. This thermocouple is used to measure the surface temperature. The temperature inside the blade was measured with a Type K thermocouple glued to the metal with a metal-filled epoxy. See Figure 3.

Table 1 Comparison of Properties between the Model Blade and the Thermocouple Tube.

	Blade	Thermocouple Tube
Aluminum Alloy	6061	3003
Density, kg/m^3	2700	2720
Conductivity, W/m-K	155-180	162
Specific Heat, J/kg-K	896	893

Conventional thermocouples usually make use of an electronic ice bath built into the computer. In this case, that was not possible because of the unusual thermocouple metal pair used, therefore a real ice bath was used to create the reference temperature of 0 ^{0}C.

Measurements were made at a frequency of 100 Hz.

Results

Details of the test conditions and procedures can be found in Popp et al.[16]

In Figure 5 below, typical data for the measured temperature on the surface as given by our Aluminum/Constantan thermocouple and the measured temperature on the back face are given as a function of time.

The HFS heat flux gage is also able to measure a temperature using a platinum resistance thermometer (RTS). When comparing the RTS and the Aluminum thermocouple temperatures, one sees only a slight difference as shown in Figure 6. A possible error might be a delay in the RTS temperature, because of the larger thermal mass associated with the heat flux gage. The response times of the coaxial thermocouple are very fast, and claims by Medtherm are in the order of 1 μs.

Now, we will present and discuss heat flux results for this test obtained by several different methods. First, the heat flux measured by the HFS gage is presented in Figure 7 below. Data acquisition started at time zero, the moment heated air started to flow. The curve shows measurable values of heat flux after approximately 3 seconds. The maximum value is approximately 5 W/cm^2 at 4.2 sec.

Second, we can use the measured surface temperature time history from the coaxial thermocouple with the Cook-Felderman technique. Recall that this technique cannot take advantage of the measured backface temperature, since it assumes a semi-infinite body. The deduced heat flux in Figure 8 is the outcome. The grey signal is the original and the heavy black is a ten-point running average. The first thing one can note is the noisy result. The Cook-Felderman technique greatly amplifies any noise in the surface temperature signal.

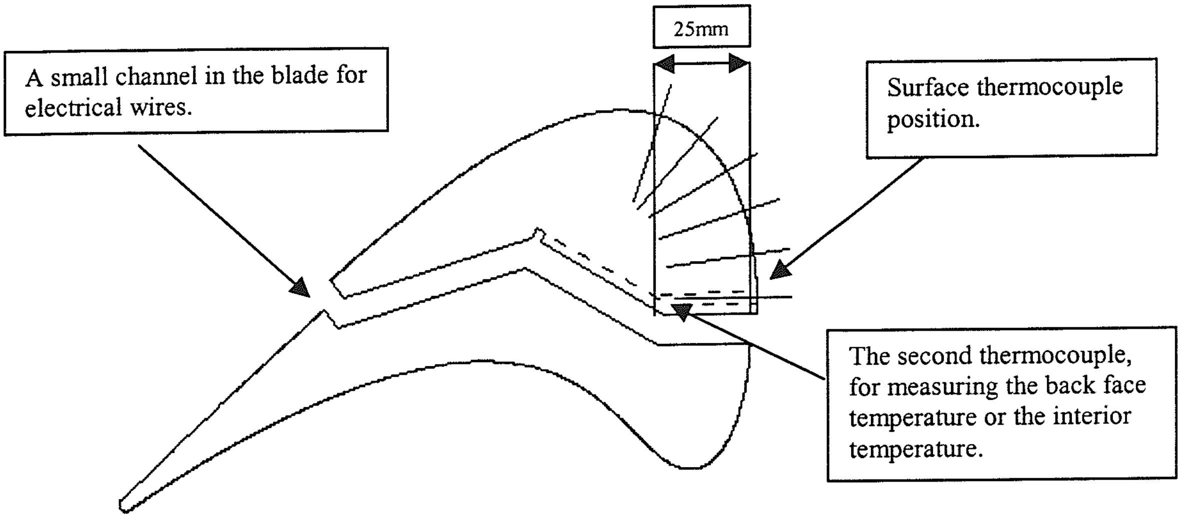

Figure 3. The Blade Showing the Two Thermocouple Positions.

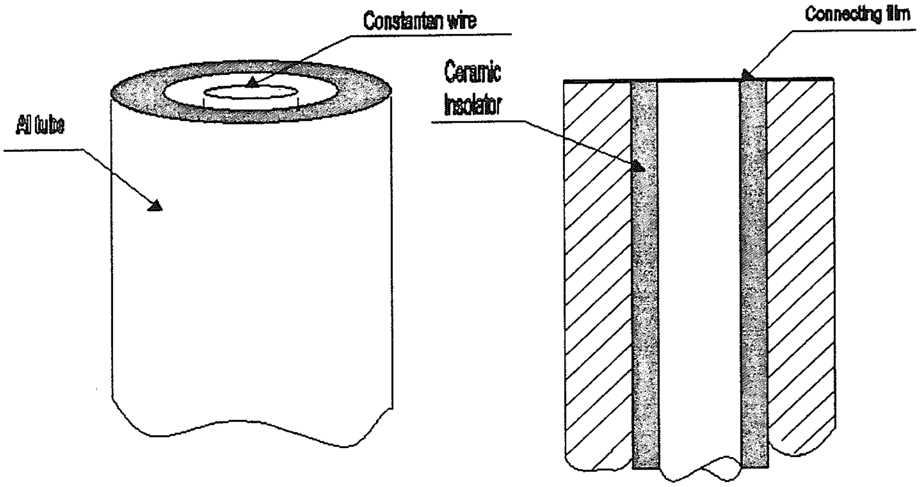

Figure 4. Aluminum-Constantan Coaxial Thermocouple.

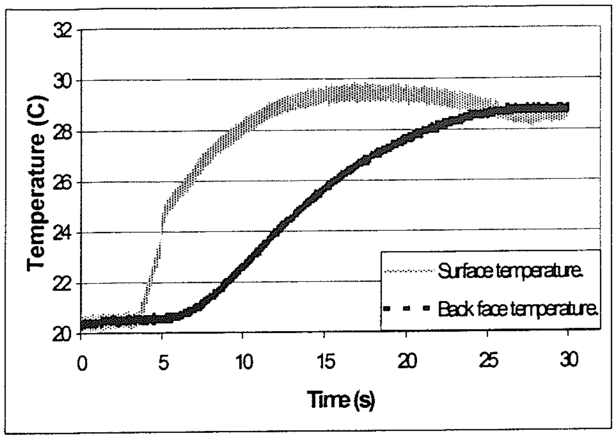

Figure 5. Typical Measured Surface and Backface Temperature Histories.

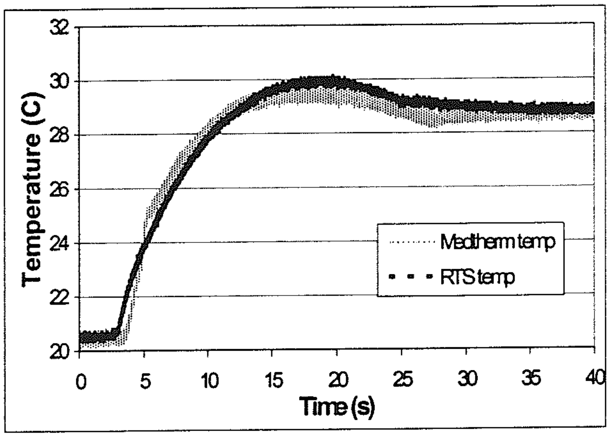

Figure 6. Comparison of the RTS and Coaxial Thermocouple Surface Temperatures Histories.

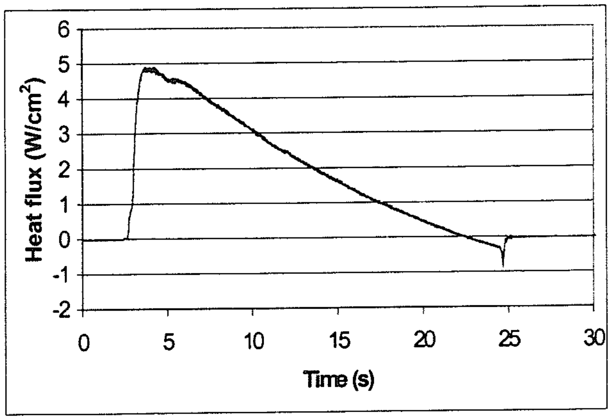

Figure 7. Heat Flux Results from the HFS Gage.

Next, consider the results as represented by the average curve. The sharp initial peak in the heat flux is due to the sharp increase in the temperature at this point measured by the coaxial thermocouple, indicating the rapid starting process of the facility. If this initial spike is ignored for the moment, the maximum value of the flux is on the order of 6 W/cm^2, which can be compared to the HFS maximum of 5 W/cm^2. This does not mean that the spike can be ignored when we look at the global picture. The HFS gage might smear this spike, because of its larger "footprint" and likely slower response time than the coaxial thermocouple. One can expect that the flux deduced at later times (after about 12 sec.) using the Cook-Felderman technique is higher than the actual value, because the internal temperature is rising compared to the semi-infinite solid case as assumed in the Cook-Felderman method. In the semi-infinite solid case, the temperature in the middle of the blade is assumed to be constant. In reality this is not the case. The temperature does rise, and the slope of the temperature profile will be slightly less, causing the flux to be less as well.

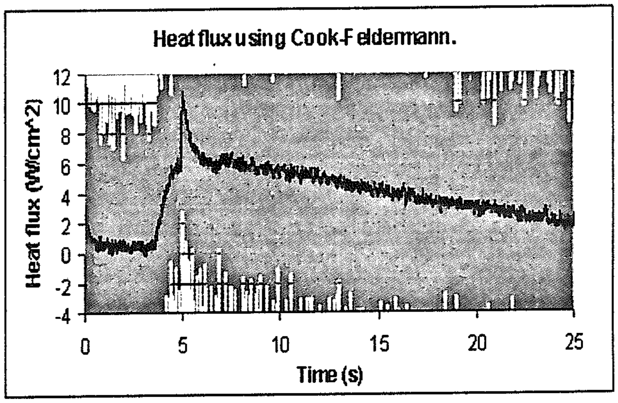

Figure 8. Heat Flux Predicted with the Cook-Felderman Technique.

Third, the measured surface and backface temperature histories can be used to deduce the heat flux with a simple implicit finite difference (Class 2) method. This is a normal finite difference method, using the two measured temperatures as the unsteady boundary conditions. In this and the next approaches, the time step and the spacing were the same: $\Delta x = 0.25$mm and $\Delta t = 0.01$s. The heat flux results are shown in Figure 8. It is clear that this method also amplifies the noise in the measured temperature, but not nearly as severely as with the Cook-Felderman method. The running average curve displays many of the features shown in Figure 9 for early times (less than about 12 secs.) where they should be comparable. The sharp peak is captured, and a maximum value after that peak of about 6 W/cm^2 is indicated. The heat flux values in Figure 9 after about 12 secs. are lower than in Figure 8, as they should be since the actual backface temperature variation is now included.

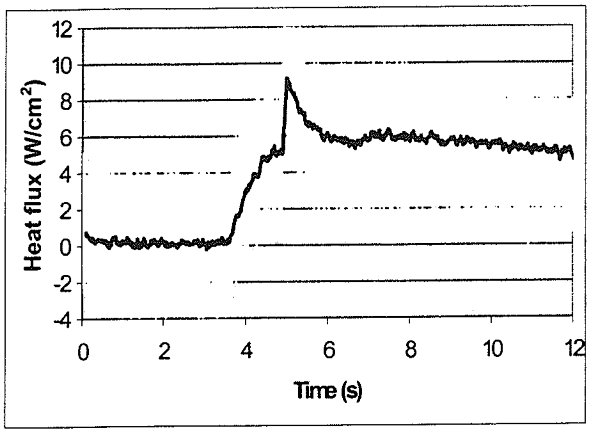

Figure 9. Heat flux Predicted with an Implicit Finite Difference (Class 2) Technique.

Last, the measured surface and backface temperature histories can be used to deduce the heat flux with the extended inverse method code, and the results are presented in Figure 10. The most striking thing one can see is that the noise in the temperature signal is amplified to a much smaller degree than with either the Cook-Felderman method (Fig. 8) or the numerical method (Fig. 9). The running average curve is very similar to the one in Figure 9 obtained with the inverse method and the early time (less than 12 sec.) results in Figure 8 obtained with the Cook-Felderman method. The maximum value after the sharp initial peak is again approximately 6 W/cm^2 compared to the value of 5 W/cm^2 from the HFS gage.

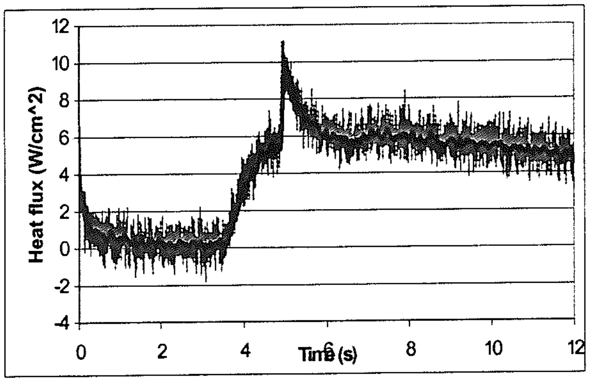

Figure 10. Predicted Heat Flux from the Extended Inverse Method Code.

Uncertainty Analysis

We undertook an uncertainty analysis for the methods used in this work. In addition to the usual uncertainties associated with instruments, several other possible sources of uncertainty were considered.[18]

One potential cause for uncertainty is that the heat flux is assumed to be 1-D in all the methods used except for the HFS gage. Data concerning the variation of the heat flux along the model blade surface from Popp et. al.[16] was used with an idealized representation of the aluminium model blade in a 2-D heat conduction numerical method to evaluate this matter. The results demonstrated that any errors from 2-D effects were negligible in this experiment.[18]

For the experiment, an ice-bath was used for the reference temperature of 0°C. An error here would alter the temperature measured with the thermocouple in the blade. Good accuracy was achieved by inserting this reference thermocouple into a glass tube, filled with oil and then placed into a thermos flask. If such a technique is used, the uncertainty in the reference junction temperature can be made negligibly small.[19]

The calibration factor for the Aluminum/Constantan coaxial thermocouple was taken as 39.6 μV/°C. This value was supplied by the Medtherm corporation. To verify this calibration factor, a number of points of temperature versus voltage output were measured. The slope of a linear fit through the data, revealed a value of 40.7 μV/ °C . This is quite close to the value supplied by the manufacturers of the thermocouple. The maximum error from this source was estimated as 3%.[18]

The Type K thermocouple used for the backface temperature might also introduce some error in the final result. There are basically two potential problem areas. First, the calibration of the thermocouple can be imperfect. The values used are built into the LabView program. Type K thermocouples are widely used in the industry, and calibration factors for them are very accurate. This problem is thus not a dominant one. Second, the fact that the thermocouple is glued to the aluminum surface creates some uncertainty. This uncertainty was minimized by using an Aluminum based metal epoxy. These epoxies conduct heat very well, so the assumption that the thermocouple is actually part of the solid was made.

The properties of most metals change with a change in the temperature. Here, the assumption was made to keep the properties constant, because of the small range of the temperature encountered. Table 1 shows the range of the thermal conductivity. It also shows the difference in the properties of the blade and the thermocouple. All of these differences create a margin of error. Calculations were redone where the conductivity was changed to the minimum value of 155 W/mK. The results were quite close to the earlier calculations where a conductivity value of 180 W/mK was used (approximately 4% difference).[18]

Finally, one can use Duhamel's method[20] for the integration of the deduced heat flux values to back-calculate the temperature histories, and those can be compared to the actual measured temperature histories. This provides a useful check on the internal consistency and accuracy of the various methods. For simplicity, we restricted the application of Duhamel's method to semi-infinite cases, therefore it is expected that the results will diverge at larger times (greater than about 12 secs.). The calculations were made for the Cook-Felderman technique and the inverse method as well as the measured flux from the HFS heat flux gage, and results in Figure 11 were obtained. The measured temperature curve is obscured by the results from the Cook-Felderman method. The results indicate the heat flux values determined from the Cook-Felderman technique and the inverse method are consistent with the measured surface temperature variation. The heat flux values from the HFS gage are not consistent with the measured surface temperature variation on that gage, which was very close to that indicated by the coaxial thermocouple (see Fig. 6).

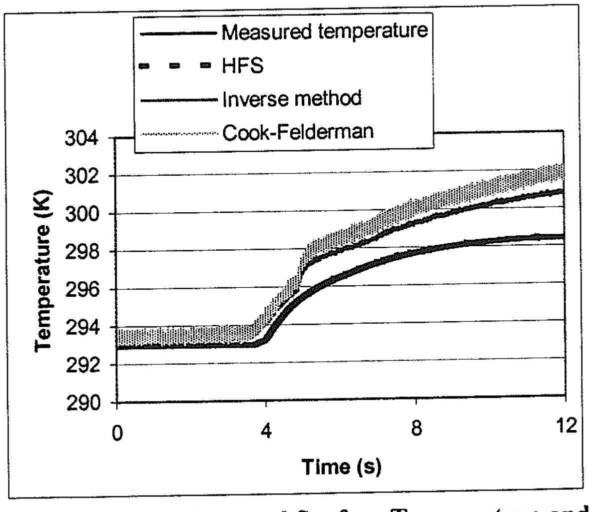

Figure 11. The Measured Surface Temperature and the Temperatures Calculated with Duhamel's Method for the Three Different Techniques.

Taking all these considerations together with the low uncertainty due to instruments, it our judgement that the present procedure is accurate to within an uncertainty of approximately 3-4%.[18]

Discussion

The goal of the present work is to implement a simple, cost-effective method of obtaining the surface heat flux to an object exposed to a high-speed flow with a small, but robust, sensor. In this particular case, the long-term goal was to install this sensor into an existing skin-friction gage. An extensive literature study was done in order to pursue this need. Eventually, the decision was made to make use of a coaxial thermocouple installed to measure the surface temperature as a function of the time. A second thermocouple was installed inside the solid body so that the common semi-infinite body assumption could be relaxed. To insure similar heat flux through the object and the heat flux sensor, the material of the thermocouple can be chosen in such a way that the material properties will be very similar to that of the object. Both of these temperature profiles were then converted to surface heat flux through the use of an inverse method.

The inverse approach was chosen because of its stability and the fact that the other methods available in the literature amplify measurement noise in the temperature signal. An existing code for the inverse method was used as the starting point for this effort. A custom forward model was employed by the inverse method to provide temperature calculations. The computational time is more than that required by the Cook-Felderman technique, but the estimates from the inverse technique were deemed superior for applications of this type. In addition, the Cook-Felderman technique, at least as presently used, is restricted to semi-infinite bodies, and that is often a severe limitation in testing. The inverse method offers a simple, yet accurate way of obtaining the heat flux at the surface of an instrumented object.

An experimental trial application was used in the verification of the method. This experiment involved the measurement of heat flux through a model turbine blade in a transonic cascade tunnel. Comparisons were made to the HFS heat flux gages installed in the blades and the outcome was quantified. In the experiments, the heat flux from the inverse method using surface and backface thermocouples was about 20% higher than that of the HFS gage. However, the temperature residuals indicate that the fluxes estimated by the inverse procedure more closely match the measured temperatures. Furthermore, the HFS gages used by the other research groups working in the same field, do not have the advantage of simplicity and robustness offered by the coaxial thermocouple as used in this work. Also, such gages require calibration, which is neither simple nor accurate for high heat fluxes. Possible sources of errors with the current method were examined in detail and found to be about 3-4%.

Another advantage of the proposed technique is that the measuring devices can be made very small. This will then be suitable for very confined spaces. It is important to keep this in mind, because the size of any measuring device may play a vital role in certain applications, such as confined spaces and weight limitations. In addition, the footprint of the heat flux measuring devices can be made very small, because the head of the thermocouple can be made very small. In other words the flux can be said to have a certain value at a specific point on the surface, rather than is the case with the HFS gages, where the flux has some value over a small region on the surface.

Since measurement of temperature is on the surface, the standard inverse method can be simplified by using a single time step. Walker[13] experimented with this method and obtained good results. He called the modified regularization procedure the *Explicit-Regularization* method. This method yields the same numerical flux values as the implicit method, but convergence is much faster.

Bibliography

1) Schetz, J.A., "Direct Measurements of Skin Friction in Complex Flows," Appl. Mech. Rev., Vol. 50, No. 11, Pt. 2, Nov. 1997.

2) Diller, T.E., "Advances in Heat Flux Measurements," in: Advances in Heat Transfer, Vol. 23, Eds. J.P. Hartnett et al., Academic Press, Boston, pp. 279–368, 1993.

3) Diller, T. E., and Kidd, C. T., "Evaluation of Numerical Methods for Determining Heat Flux with a Null Point Calorimeter," Proceedings of the 42nd International Instrumentation Symposium, ISA, Research Triangle Park, NC, pp.251-262, 1997.

4) Cook, W. J., and Felderman, E. J., "Reduction of data from Thin Film Heat Transfer Gages: A Concise Numerical Technique," AIAA Journal, Vol. 4, No. 3, pp. 561-562, 1996.

5) William, K.G., William, J.R., and Scott, H. W., "An Evaluation of Analog and Numerical Techniques for Unsteady Heat Transfer Measurement with Thin-film Gages in Transient Facilities," Calspan-UB Research Center, Buffalo, New York, Elsevier Science Publishing, New York, 1991.

6) Beck, J. V., Blackwell, B., and St. Clair, C. R., Jr., Inverse heat conduction: Ill posed problems, John Wiley & Sons, New York, 1985.

7) Ozisik, M.N.,Boundary Value Problems of Heat Conduction, Constable and Company, Ltd., London, 1968.

8) Cook, W. J., "Determination of Heat Transfer Rates from Transient Surface Temperature Measurements," AIAA Journal, Vol. 8, No. 7, pp. 1366-1368, 1970.

9) Cook, W. J., "Unsteady Heat Transfer to a Semi-Infinite Solid with Arbitrary Surface Temperature History and Variable Thermal Properties," Tech. Report, Engineering Research Institute TR ISU-ERI-Ames 67500, Iowa State University, Ames, Iowa, 1970.

10) Diller, T. E., "Methods of Determining Heat Flux from Temperature Measurements," Department of Mechanical Engineering, Virginia Tech, Blacksburg, Virginia, pp. 357-369, 1996.

11) Ehrich, E.F., "Differentiation of Discrete Data," Journal of Aeronautical Sciences, Vol. 22, p. 133, 1954.

12) Tikhonov, R.E., and Arsenin, V.Y., Solutions of Ill-Posed Problems, V.H. Winston & Sons, Washington D.C., 1977.

13) Walker, G., "Estimation of Unsteady Nonuniform Heating Rates from Surface Temperature Measurements," Ph.D. Dissertation, Virginia Polytechnic Inst. and State Univ., 1997.

14) Scott, E.P. and Beck, J.V., "Analysis of Order of Regularization," Journal of Heat Transfer, Vol. 111, p. 218, 1987.

15) Walker, D.G. and Scott, E.P., "Evaluation of Estimation Methods for High Unsteady Heat Fluxes from Surface Measurements," Journal of Thermophysics and Heat Transfer, Vol. 12, p. 543, 1998.

16) Popp, O., Smith, D.E., Bubb, J.V., Grabowski, H.C., Diller, T.E., Schetz, J.A. and Ng, W.F., "Steady and Unsteady Heat Transfer in a Transonic Film Cooled Turbine Cascade," ASME Paper No. 99-GT-259, 1999.

17) Kidd, C.T., Nelson, C.G. and Scott, W.T.,"Extraneous Thermoelectric EMF Effects

Resulting from the Press-Fit Installation of Coaxial Thermocouples in Metal Models," Instr. Soc. America, Paper 94-1022, 1994.

18) Bezuidenhout, J., "Convective Heat Flux Determination using Surface Temperature History Measurements and an Inverse Calculation Method, " MS Thesis, Virginia Polytechnic Institute and State University, 1999.

19) ASTM Committee E20 on Temperature Measurement, Manual on the use of thermocouples in temperature measurement, Fourth Edition, Philadelphia, PA., 1993.

20) Carslaw, H. S., and Jaeger, J.C., Conduction of Heat in Solids, 2nd ed., Oxford University Press, London, pp85-88, 1959.